Space-time Interaction Principle as a Description of Quantum Dynamics of Particle

Chu-Jun Gu

Institute of Theoretical Physics, Grand Genius Group, 100085, Beijing, China

Abstract

We propose a space-time interaction principle (StIP) which states any particle with mass m will involve a random motion without friction, due to random impacts from space-time. Every impact changes the amount $\hbar$ for an action of the particle. According to the principle, firstly, we prove the interaction coefficient must be $\Re = \frac{\hbar}{2m_{ST}}$ deriving from Langevin's equation to the corresponding Fokker-Planck Hamiltonian, where m_{ST} is a space-time sensible mass of the particle. We can derive that an equation of motion for the particle will be the Schrödinger equation, and prove that the space-time sensible mass m_{ST} reduce to the inertial mass in the non-relativistic quantum mechanics. Secondly, we show that there must exist the smallest mass $\bar{m}_{ST}$ as the minimum of space-time sensible mass, provided the speed of light in vacuum as the maximum speed due to the postulation of special relativity. Furthermore, we estimate a magnitude of this $\bar{m}_{ST}$ from microwave background radiation. Thirdly, an interpretation of Heisenberg's uncertainty principle is suggested, with a stochastic origin of Feynman's path integral formalism. It is shown that we can construct a physical picture distinct from Copenhagen interpretation, and reinvestigate the nature of space-time and reveal the origin of quantum behaviours from the materialistic point of view.

I. COMMENTS ON THE COPENHAGEN INTERPRETATION

The main idea of Copenhagen interpretation is that the wave function does not have any real existence in addition to the abstract concept. Whether the wave function is an independent entity, the Copenhagen interpretation does not make any statement. In this article we do not deny the internal consistency of Copenhagen interpretation. We admit that Copenhagen's quantum mechanics is a self-consistent theory. Einstein believed that for a complete physical theory, there must be such a requirement: a complete physical theory should include all of the physical reality, not merely its probable behaviour. From the materialistic point of view, the physical reality should be measured in principles , such as the position q and momentum p of particles.

In the Copenhagen interpretation, the particle wave function $\Psi(q,t)$ or the momentum wave function $\Psi(p,t)$ is taken to be the only description of the physical system, which can not be seen as a complete physical theory[1] but only a phenomelogical effective theory. Therefore, in this paper, we propose an StIP where the coordinate and momentum of particles are objective reality not affected by the observations.With the postulation of StIP, quantum behaviour will emerge from a statistical description of space-time random impacts on the experimental scale, including Schr$\ddot{o}$dinger equation, Born rule, Heisenberg's uncertainty principle and Feynman's path integral formulation. Thus, we believe that non-relativistic quantum mechanics can be constructed under the StIP. In this picture, the Born's probability description for quantum mechanics as well as Heisenberg's uncertainty principle do not have meanings of fundamental principles, rather that they can be derived from the StIP as emergent phenomena.

II. THE ORIGIN OF THE STIP COEFFICIENT

Definition 1. The quanta of space-time (QST) is the smallest element unit of quantized space-time.

Proposition 2. *Massive particles in space-time will be random collided by quantas of space-time (QSTs), the change of action during each collision is exact $\hbar$,the reduced Planck constant. The motion of the particle under this action is a Markov process.*

Let m_{ST} be the mass of the particle detected by space-time. We call m_{ST} the space-time sensible mass. We will prove the space-time interaction coefficient of a mass m_{ST} particle will be universally given as

$$\Re = \frac{\hbar}{2m_{ST}}. \tag{II.1}$$

Within the framework of random motion[2], or Wiener process in mathematics [3], this space-time induced random motion is equivalent to the Markov process, moreover, the space-time interaction coefficient is nothing but the diffusion coeffient [4]. In this section, we will start our journey from propability theory of random motion[4, 5], and then give a concrete proof that for the random motion induced by StIP(StIP), the space-time interaction coefficient is given exactly by $(II.1)$. After that, following the StIP and the hypothesis of light speed, we derived that in space-time there should exists a universal minimal mass $\bar{m}_{ST}$, which is the lower bound mass can be detected by the QST. We estimate the minimal mass according to discussions of cosmological microwave background radiation[6]. The last two subsections discussed two space-time model in order to investigate the origin of the space-time interaction coefficient. From both we obtained the coefficient reads as $\Re = \frac{w\ell}{2}$, in which w is the average speed of the particle and ℓ the mean free distance.

A. Probabilitistic proof of space-time interaction coefficient

We assume in a quantized space-time, full of QSTs, which are smallest elements of this quantized space-time. A particle in space-time, naturally will suffer collisions from these QSTs, and will emerge an arbitrary random motion. Each collision can be seen as a classical elastic one, under conservation laws of energy and momentum, the particle will obtain or loss certain amount of energy and momentum. If the particle is not scattered by QSTs, it will keep in moving along a straight line with uniform velocity. When a collision happened, the state of motion of the particle changed, however, the change is not instant, since an instant collision means in physically the instant force is infinitely large. An explicit observation can be achieved by introducting the impulse-momentum theorm:

$$\vec{F}\Delta t = m\vec{V} - m\vec{V}_0 = m\Delta\vec{V} = \Delta\vec{P}. \tag{II.2}$$

1. The effect of separation of paths

At a certain moment, particle can be scattered by many QSTs with different momenta and energies. In Δt, we assume there are effectively N collisions. The state of the motion will depends on the net effect of the N times collision. This is a principle of superposition. We can use in total N vectors to superposite whole changes of the state of motion, which means if at time t the particle was at position $\vec{X}(t)$, velocity $\vec{V_0}$, then at the moment $t+\Delta t$, its position will be $\vec{x}(t+\Delta t) = \vec{X}(t) + \sum_{i=1}^{N} \Delta X_i$, and velocity$\vec{V_0} + \sum_{i=1}^{N} \Delta \vec{V_i}$. This simple analysis tells us in Δt, the ultimate state of motion of the particle can be separated as N different paths. This is the effect of separation of paths. While the weights of these paths, aka the probability distribution of universal diffusion, highly relies on the energy distribution of QSTs. That is saying, collisions by QSTs with different energies end up with different changes of the state of motion.

2. Minimal energy

If the scattering QST has an extremely low energy such that in Δt, the transfered action is less then $\hbar$, we conclude that in Δt, the QSTs cannot collide the particle. Classically, we argue that such a collision still in process, the particle as well as the QST are in a bound state, not a scattering state. This is similar to completely inelastic collision in classic mechanics, while in such a process, the conservation of energy and momentum can not be satisfied simultaneously. Because of conservation of energy and momentum, the bound state actually is not a stable state. The bounded particle is a virtual particle. This observation leads to an important point: there exists a minimal energyE_{min} in Δt so that

$$E_{min}\Delta t = const. \tag{II.3}$$

In physics , the product of energy and time will have the unit of action. It is natural to conjecture such a constant with action unit is the Planck constant, so we have

$$E_{min}\Delta t = \hbar. \tag{II.4}$$

3. Langevin Equation

We argue, the energy distribution of the QSTs in space-time, will be in type of Gaussian. For one thing this is ensured by the central limit theorem in probability theory [5]. For another, if the distribution is not Gassian, then the particle in space-time will have no reason to act randomly. The Gaussion distribution reflects the QSTs are universal white noises for particles in space-time. The space-time background can be seen as a enviroment of white noise, while the particle is moving under the interaction of the enviroment, its motion is described by a Markov process. The corresponding movement can be determined by the Langevin equation[7]:

$$\frac{dq_i(t)}{dt} = -\frac{1}{2} f_i(\mathbf{q}(t)) + \nu_i(t) \tag{II.5}$$

where $q_i(t)$ describes the trajectory of the particle, and $f_i(\mathbf{q})$ is a differentiable function, which captures the classical motion of the particle. The ν_i is a white noise, here means the interaction function induced by QSTs.For a Markov process, the average contribution of white noise vanishes. However, because of its Gaussian nature, its variation is not zero, we have

$$\langle \nu_i \rangle_\nu = 0,\ \langle \nu_i(t)\nu_j(t') \rangle_\nu = \Omega \delta_{i,j} \delta(t-t') \tag{II.6}$$

here the$\delta_{i,j}$ in the later equation can be obtained from the space-time homogeneous property, while $\delta(t-t')$ determined from the Markov property. For a Markov process, only conditions at the moment determine the dynamics of the system, all information from future or past are irrelevant. We can write down the basic correlation function by introducing a probability measure $[d\rho(\nu)]$,which is given as

$$[d\rho(\nu)] := \left(\sqrt{\frac{\Omega}{2\pi}} \right)^D [d\nu] \exp\left(-\frac{1}{2\Omega} \int dt \sum_i \nu_i^2 \right) \tag{II.7}$$

It is easy to see that

$$\int \nu_i(t)[d\rho(\nu)] = 0 = \langle \nu_i(t) \rangle_\nu \tag{II.8}$$

$$\int \nu_i(t)\nu_j(t')[d\rho[\nu]] = \Omega \delta_{i,j} \delta(t-t') = \langle \nu_i(t)\nu_j(t') \rangle_\nu \tag{II.9}$$

here Ω describes the strength of space-time interaction on the particle. However, from the definition of measure $(II.7)$, we can see, ν_i have the unit of m/s, so Ω will have the unit of m^2/s. From previous analysis, each collision leads to a change of an action $\hbar$. $\hbar$ has the unit of angular momentum, $kg \cdot m^2/s$. From this we can define a quantity with mass unit, it is

$$m_{ST} \equiv \frac{\hbar}{\Omega}. \tag{II.10}$$

The mass m_{ST} has the meaning such that it is the mass detected by QSTs, which is named as space-time sensible mass in this article. Accordingly, the collision parameter $\Omega = \frac{\hbar}{m_{ST}}$. From which we could read off the result of the random collison, in the sense of physical realistic viewpoint. Because for an object in our real nature, the larger its mass means the smaller its quantum effect.

Langevin equation generates a probability such that

$$\mathbf{P}[\mathbf{q}, t;\ \mathbf{q_0}, t_0] = \langle \prod_{i=1}^{D} \delta[q_i(t) - q_i] \rangle_\nu,\ t \geq t_0 \tag{II.11}$$

which means for an operator $\mathcal{O}[\mathbf{q}]$, its averate value at time t will be:

$$\langle \mathcal{O}[\mathbf{q}(t)] \rangle_\nu \equiv \int \mathbf{P}[\mathbf{q}, t;\ \mathbf{q_0}, t_0] \mathcal{O}[\mathbf{q}] d\mathbf{q} \tag{II.12}$$

Using the probability distribution $(II.11)$, one can immediately verify equation $(II.12)$. Actually, the distribution $(II.11)$ can be seen as an evolution process, which says

$$\mathbf{P}[\mathbf{q}, t;\ \mathbf{q_0}, t_0] = \langle q | e^{-H(t-t_0)} | q_0 \rangle$$

here the evolution Hamiltonian is the famous Fokk-Planck Hamiltonian, as we will derive its formalism in next subsection.

4. Fokk-Planck equation

Given the Langevin equation $(II.5)$, we can derive the corresponding Fokk-Planck equation, as well as the Fokk-Planck Hamiltonian [4].

We consider the time segment from t to $t + \epsilon,\ \epsilon \to 0$, we have the Langevin equation as:

$$q_i(t + \epsilon) - q_i(t) = -\frac{1}{2}\epsilon f_i(\mathbf{q}(t)) + \int_t^{t+\epsilon} \nu_i(\tau) d\tau + O(\epsilon^{3/2}) \tag{II.13}$$

its related propability distribution is

$$\mathbf{P}[\mathbf{q}, t + \epsilon;\ \mathbf{q}', t] = \langle \delta(\mathbf{q} - \mathbf{q}(t + \epsilon)) \rangle_\nu \tag{II.14}$$

To obtain the Fokk-Planck equation, we define following discreterization

$$\sqrt{\epsilon}\bar{\nu}_i := \int_t^{t+\epsilon} \nu_i(\tau)d\tau$$

so that the discrete Langevin equation is

$$q_i(t+\epsilon) - q_i(t) = -\frac{1}{2}\epsilon f_i(\mathbf{q}(t)) + \sqrt{\epsilon}\bar{\nu}_i + O(\epsilon^{3/2}) \tag{II.15}$$

Notice here the time has been discreterized as

$$(t-t')/\epsilon \in \mathbb{Z}^+.$$

Now the Gaussian distribution and the property of Markov progcess determins the average value of discrete white noises ν_i, we have

$$\langle \bar{\nu}_i \rangle_\nu = 0,\ \langle \bar{\nu}_i(t)\bar{\nu}_j(t') \rangle_\nu = \frac{\hbar}{m_{ST}}\delta_{i,j}\delta_{t,t'} \tag{II.16}$$

when $\epsilon \to 0$, the Fourier expansion of the probability distribution $(II.14)$ is

$$\begin{aligned}
\tilde{\mathbf{P}}[\mathbf{p}, t+\epsilon;\ \mathbf{q}', t] &= \int e^{-i\mathbf{p}\cdot\mathbf{q}}\mathbf{P}[\mathbf{q}, t+\epsilon; \mathbf{q}', t]d^D\mathbf{q} \\
&= \langle e^{-i\mathbf{p}\cdot\mathbf{q(t+\epsilon)}} \rangle_\nu \\
&= \langle e^{-i\mathbf{p}\cdot(\mathbf{q(t)}+\epsilon\frac{\mathbf{dq(t)}}{\mathbf{dt}}+\mathbf{O}(\epsilon^{\mathbf{2}})} \rangle_\nu \\
&= \langle \exp(-i\mathbf{p}\cdot(\mathbf{q}'(\mathbf{t}) - \epsilon/\mathbf{2f}(\mathbf{q}'))) \rangle_\nu \\
&\quad \times \left\langle \exp\left[-i\mathbf{p}\,\centerdot \int_t^{t+\epsilon} \nu(\tau)d\tau\right] \right\rangle_\nu \times \left\langle \exp\left(O(\epsilon^2)\right) \right\rangle_\nu \\
&= \exp\left[-i\mathbf{p}\,\centerdot(\mathbf{q}' - \epsilon f(\mathbf{q}')/2)\right] \\
&\quad \times \left\langle \exp\left[-i\mathbf{p}\,\centerdot \int_t^{t+\epsilon} \nu(\tau)d\tau\right] \right\rangle_\nu
\end{aligned} \tag{II.17}$$

Notice that the last average value can be evaluated out by Gaussian integration, which reads,

$$\left(\sqrt{\frac{\hbar}{2\pi}}\right)^D \int [d\nu] \exp\left(-\frac{m_{ST}}{2\hbar}\int dt \sum_i^D \nu_i^2\right) \exp\left[-i\mathbf{p}\cdot\int_t^{t+\epsilon}\nu(\tau)d\tau\right] \tag{II.18}$$
$$= \left(\sqrt{\frac{\hbar}{2\pi}}\right)^D \int [d\nu] \exp\left(-\frac{m_{ST}}{2\hbar}\int dt \sum_i \nu_i^2 - i\mathbf{p}\cdot\int_t^{t+\epsilon}\nu(\tau)d\tau\right)$$
$$= \left(\sqrt{\frac{\hbar}{2\pi}}\right)^D \int [d\nu] \exp\left(-\frac{m_{ST}}{2\hbar}\int dt \sum_i \nu_i^2 - i\sqrt{\epsilon}\mathbf{p}\cdot\bar{\nu}\right)$$
$$\times \exp\left(+\epsilon\frac{\hbar}{2m_{ST}}\mathbf{p}\cdot\mathbf{p} - \epsilon\frac{\hbar}{2m_{ST}}\mathbf{p}\cdot\mathbf{p}\right)$$
$$= \left(\sqrt{\frac{\hbar}{2\pi}}\right)^D \int [d^N\left(\nu_i + \frac{i\hbar}{2m_{ST}}\sqrt{\epsilon}p_i\right)]$$
$$\times \exp\left(-\frac{m_{ST}}{2\hbar}\int dt \sum_{i=1}^D\left(\nu_i + \sqrt{\epsilon}\frac{i\hbar}{2m_{ST}}p_i\right)^2 - \epsilon\frac{\hbar}{2m_{ST}}\mathbf{p}\cdot\mathbf{p}\right)$$
$$= \exp\left(-\epsilon\hbar\mathbf{p}\cdot\mathbf{p}/2m_{ST}\right)$$

here we can obtain the Fourier expansion of the probability distribution,

$$\tilde{\mathbf{P}}[\mathbf{p}, t+\epsilon;\ \mathbf{q}', t] = e^{-\epsilon\hbar/2m_{ST}\mathbf{p}\cdot\mathbf{p} + i\epsilon\mathbf{p}\cdot f(\mathbf{q}')/2 - i\mathbf{p}\cdot\mathbf{q}'} \tag{II.19}$$

for $\epsilon \to 0$, expanding $(II.19)$ will end up with

$$\tilde{\mathbf{P}}[\mathbf{p}, t+\epsilon;\ \mathbf{q}', t] = e^{-i\mathbf{p}\cdot\mathbf{q}'}(1 - \epsilon H_{FP}(\mathbf{p}, \mathbf{q}') + O(\epsilon^2)),$$

here we obtained the celebrate Fokk-Planck Hamiltonian

$$H_{FP}(\mathbf{p},\mathbf{q}) = -\frac{\hbar}{2m_{ST}}\mathbf{p}\cdot\mathbf{p} - i\mathbf{p}\cdot f(\mathbf{q}) \tag{II.20}$$

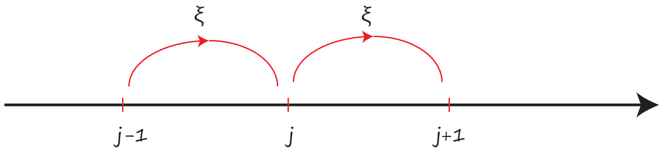
From which we can read off the diffusion coefficient induced by collisions between QSTs and the particle, is exactly $\Re = \hbar/2m_{ST}$. Later we will see in deriving the Schrodinger equation of free particle in space-time, the space-time sensible mass m_{ST} will be identified with the inertial mass, in the framework of non-relativistic quantum mechanics.

B. From discrete space-time to the space-time interaction coefficient

Beginning with StIP, we want to investigate the origin of space-time interaction coefficient. Within the framework of discrete space-time, space-time interaction coefficient

$\Re = \frac{\hbar}{2m_{ST}}$ should be derived in terms of parameters of discrete space-time. Let us consider the simplest discrete model (see Fig.3.1), where the length union of discrete space is ℓ. $P(j,t)$ is the probability of a particle at lattice site j at time t. Because of the discrete

Figure II.1: Random jumping model on one dimensional lattice



nature of the space, all jumpings can only happen between nearest pair of positions. Given the rate of jumping between the nearest neighbour ζ and the isotropy of frictionless space, the evolution of probability should be

$$\partial_t P(j,t) = \zeta(\frac{1}{2}P(j-1,t) + \frac{1}{2}P(j+1,t) - P(j,t)) \tag{II.21}$$

the first two terms of RHS of $(II.21)$ describe the fact that jumping toward and backward from neighbors $j-1-th$ and $j+1-th$ positions respectively, have the same probability, which is $1/2$, the third term remarks the probability from $j-th$ position to neighbors. Introducing the fundamental spacing of the lattice ℓ, the eq.$(II.21)$ goes to

$$\partial_t P(j,t) = \frac{\zeta\ell^2}{2}(\frac{\frac{P(j+1,t)-P(j,t)}{\ell} - \frac{P(j,t)-P(j-1,t)}{\ell}}{\ell}) \tag{II.22}$$

In the continuum limit of space-time, which says $\ell \to 0$, and $\zeta \to \infty$, but keeping the quantity $\zeta\ell^2$ unchanged, the probability $P(j,t)$ now becomes the probability density $\rho(x,t)$, the RHS of $(II.21)$ becomes the definition of second derivative. Thus we have

$$\partial_t \rho(x,t) = \frac{\zeta\ell^2}{2}\partial_x^2 \rho(x,t). \tag{II.23}$$

It is straightforward to generalise above equation onto three dimension case, we have,

$$\partial_t \rho(\vec{r},t) = \frac{\zeta\ell^2}{2}\nabla^2 \rho(\vec{r},t) \tag{II.24}$$

Comparing with Einstein's diffusion equation[10]

$$\partial_t \rho(\vec{r},t) = \Re\nabla^2 \rho(\vec{r},t) \tag{II.25}$$

the microscopic origin of space-time coefficient will be

$$\Re = \frac{\zeta\ell^2}{2} \tag{II.26}$$

Furthermore, we can also discrete time with union $\tau = \frac{\ell}{w}$, where w is the average velocity of particle. With $\zeta = \frac{1}{\tau}$, we obtain

$$\Re = \frac{w\ell}{2} \tag{II.27}$$

Combining the microscopic structure of discrete space-time with the StIP, we have

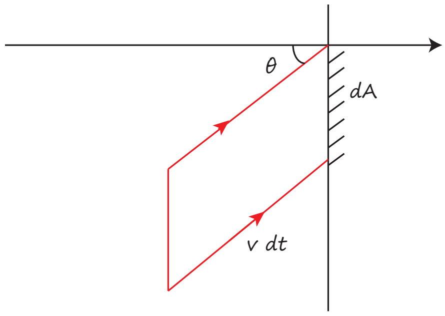
$$\Re = \frac{w\ell}{2} = \frac{\hbar}{2m_{ST}} \tag{II.28}$$

It's crucial to notice that massive particle can not exist without space-time, which should be considered as excitations of the structure of space-time. Different structures of space-time (aka different spacing ℓ) lead to different masses of particles.

C. From space-time scattering to the space-time interaction coefficient

Particles will be scattered randomly from the QST with the speed of light, which leads to the probability distribution of velocity $f(\vec{v})$. Therefore, all the particles cross the section area dA during time dt will be inside the cylinder (see Fig.3.2). The volume of this cylinder

Figure II.2: Probability distribution of space-time scattering



is

$$V = vdt\cos\theta dA \tag{II.29}$$

in which the number of particles is

$$N = f(\vec{v})d^3\vec{v}vdt\cos\theta dA \tag{II.30}$$

Because of the isotropy of space, we have $f(\vec{v}) = f(v)$. From left to right, the number of particle cross the unit area per unit time is

$$\Phi = \int_{v_z>0} \frac{N}{dAdt} = \int_0^{\frac{\pi}{2}} d\theta\cos\theta\sin\theta \int_0^{2\pi} d\varphi \int_0^{+\infty} f(v)v^3dv = \pi \int_0^{+\infty} f(v)v^3dv \tag{II.31}$$

where $v_z > 0$ means $0 < \theta < \frac{\pi}{2}$. The average velocity reads

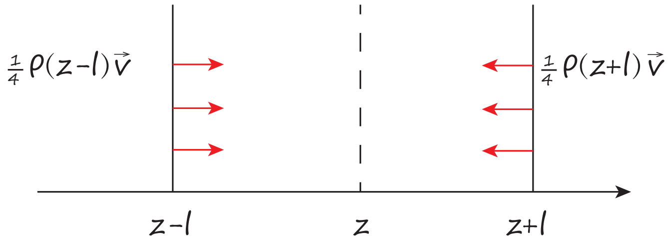
$$w = \frac{\int_0^{+\infty} f(v) v d^3v}{\int_0^{+\infty} f(v) d^3v} = \frac{4\pi}{\rho} \int_0^{+\infty} f(v) v^3 dv \tag{II.32}$$

where the density of particle number is $\rho = \int_0^{+\infty} f(v) d^3v$. Correspondingly, the number of particle cross the unit area per unit time will be

$$\Phi = \frac{1}{4}\rho w \tag{II.33}$$

Let mean free path of particles be ℓ, i.e. the average distance traveled by the particle between successive impacts from space-time. The net flux J_z through the z plane will be (see Fig.3.3)

Figure II.3: Mean free distance and scattering flux



$$J_z = \frac{1}{4}\rho(z-\ell)w - \frac{1}{4}\rho(z+\ell)w = -\frac{1}{2}\ell w \partial_z \rho \tag{II.34}$$

With the equation of continuity

$$\partial_t \rho + \nabla \cdot \vec{J} = 0 \tag{II.35}$$

and the isotropy of space, we have

$$\partial_t \rho = \frac{1}{2}\ell w \nabla^2 \rho \tag{II.36}$$

Combining the kinetics of space-time scattering with the StIP, we obtain

$$\Re = \frac{w\ell}{2} = \frac{\hbar}{2m_{ST}} \tag{II.37}$$

which is entirely consistent with Eq.II.28.

III. RANDOM MOTION OF FREE PARTICLE UNDER SPACE-TIME INTERACTION

A. From StIP to Schrödinger equation

Random motion of free particle under space-time interaction is a problem of stochastic mechanics, of which the most dramatic difference from Newtonian Mechanics is that the derivative $d\vec{x}/dt$ is not well defined[8, 9]. For a stochastic process $\vec{x}(t)$, its speed $\vec{V}$ can be understood as the sum of classical speed $\vec{v}$ and fluctuated speed $\vec{u}$

$$\vec{V} = \vec{v} + \vec{u} \tag{III.1}$$

With the time reversal transformation $\vec{x} \to \vec{x}, t \to -t$, it's shown that $\tilde{v} = -\vec{v}, \tilde{u} = \vec{u}$. Since a continues Markov process will still be a Markov process under time reversal, we can have a well defined limit $\vec{u} = 0$ as Newtonian Mechanics with

$$\vec{v} = \frac{1}{2}(\vec{V} - \tilde{\vec{V}}) \tag{III.2}$$

$$\vec{u} = \frac{1}{2}(\vec{V} + \tilde{\vec{V}}) \tag{III.3}$$

Without the interaction of space-time, the speed of particle $\vec{v}$ has to be the derivative $\vec{v} = \frac{d\vec{x}}{dt}$. Contrasting from usual Markov process, space-time random motion is frictionless, otherwise the quantum effect of a particle will decay as time going, which is obviously not the case. According to the StIP, the coordinate of a free particle is a stochastic process $\vec{x}(t)$, in which the velocity $\vec{V}$ can not be expressed in terms of $\frac{d\vec{x}}{dt}$. The velocity $\vec{V}$ should be a statistical average corresponding to a distribution $\delta\vec{x} = \vec{x}(t + \frac{1}{\omega}) - \vec{x}(t)$, at the limit of space-time collision frequency ω going to infinity. In Einstein's theory on Brownian motion, $\delta\vec{x}$ is a gaussian distribution with zero mean and variance proportional to $\frac{1}{\omega}$[10]. However, Einstein's theory cannot be correct at the limit of space-time collision frequency ω going to infinity[11, 12]. Therefore, we will construct the operator D as following, which plays the same role as $\frac{d}{dt}$ in Newtonian Mechanics. For any physical function $f(\vec{x},t)$, we have

$$\begin{aligned}
&\omega(f(\vec{x}(t+\frac{1}{\omega}),t+\frac{1}{\omega})-f(\vec{x}(t),t))\\
&=[\partial_t+\sum_i\omega(x_i(t+\frac{1}{\omega})-x_i(t))\partial_i\\
&\quad+\sum_{ij}\frac{\omega}{2}(x_i(t+\frac{1}{\omega})-x_i(t))(x_j(t+\frac{1}{\omega})-x_j(t))\partial_i\partial_j\\
&\quad+\sum_i(x_i(t+\frac{1}{\omega})-x_i(t))\partial_i\partial_t+\frac{1}{2\omega}\partial_t^2]f(\vec{x}(t),t)
\end{aligned}\tag{III.4}$$

At the limit of space-time collision frequency ω going to infinity, in terms of statistical average $< ... >$ for δx, we can define the operator D as

$$\begin{aligned}
Df(x(t),t) &= \lim_{\omega\to+\infty}\omega < f(\vec{x}(t+\frac{1}{\omega}),t+\frac{1}{\omega})-f(\vec{x}(t),t) > \\
&= (\partial_t+\sum_i V_i\partial_i+\sum_{ij}\Re_{ij}\partial_i\partial_j)f(\vec{x}(t),t)
\end{aligned}\tag{III.5}$$

where

$$\vec{V} = \lim_{\omega\to+\infty}\omega < \delta\vec{x} > \tag{III.6}$$

$$\Re_{ij} = \lim_{\omega\to+\infty}\frac{\omega < \delta x_i\delta x_j >}{2} \tag{III.7}$$

According to the StIP, the matrix of interaction coefficient is

$$\Re_{ij} = \frac{\hbar}{2m_{ij}} \tag{III.8}$$

Because of the isotropy of space, the space-time interaction coefficient will be

$$\Re_{ij} = \Re\delta_{ij} \tag{III.9}$$

which is consistent with Eq.II.28 and II.37. The operator D and its time reversal $\tilde{D}$ are

$$D = \partial_t+\vec{V}\cdot\nabla+\Re\nabla^2 \tag{III.10}$$

$$\tilde{D} = -\partial_t+\tilde{\vec{V}}\cdot\nabla+\Re\nabla^2 \tag{III.11}$$

Therefore, the real speed of particle $\vec{V}$ can be written as

$$\vec{V} = D\vec{x} \tag{III.12}$$

$$\tilde{\vec{V}} = \tilde{D}\vec{x} \tag{III.13}$$

Correspondingly, its classical speed and fluctuated speed are

$$\vec{v} = D^{-}\vec{x} \tag{III.14}$$

$$\vec{u} = D^{+}\vec{x} \tag{III.15}$$

with

$$D^{-} = \frac{1}{2}(D - \tilde{D}) \tag{III.16}$$

$$D^{+} = \frac{1}{2}(D + \tilde{D}) \tag{III.17}$$

We define the average acceleration of particles as

$$\vec{a} = D\vec{V} \tag{III.18}$$

With the invariance of average acceleration under time reversal, the average acceleration of a free particle must be zero, which can be written as

$$D^{-}\vec{v} + D^{+}\vec{u} = 0 \tag{III.19}$$

$$D^{+}\vec{v} + D^{-}\vec{u} = 0 \tag{III.20}$$

These conditions are equivalent to coupled non-linear partial differential equations as following

$$\frac{\partial \vec{u}}{\partial t} = -\Re\nabla^2\vec{v} - \nabla(\vec{u}\cdot\vec{v}) \tag{III.21}$$

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v}\cdot\nabla)\vec{v} + (\vec{u}\cdot\nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{III.22}$$

Random motions of free particles due to the random impacts of QST satisfies the Markov property if one can make predictions for the future of the process based solely on its present state just as well as one could knowing the process's full history. This is the simplest situation for random motions, since the free particle does not involve any external potential. Now, we have an initial value problem, which is to solve $\vec{u}(\vec{x},t)$ and $\vec{v}(\vec{x},t)$ given $\vec{u}(\vec{x},0) = \vec{u}_0(\vec{x})$, and $\vec{v}(\vec{x},0) = \vec{v}_0(\vec{x})$. In order to solve the coupled non-linear partial differential equations, we have to linearise it firstly. Let $\Psi = e^{R+iI}$, where

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{III.23}$$

$$\nabla I = \frac{1}{2\Re}\vec{v} \tag{III.24}$$

We can obtain

$$\frac{\partial\Psi}{\partial t} = i\Re\nabla^2\Psi \tag{III.25}$$

According to the StIP, the universal space-time diffusion coefficient is the space-time interaction coefficient $\Re = \frac{\hbar}{2m_{ST}}$. Substituting to the last equation, we will get the equation of motion of free particles as

$$i\frac{\partial\Psi}{\partial t} = -\frac{\hbar\nabla^2}{2m_{ST}}\Psi \tag{III.26}$$

which is the Schrödinger equation essentially. From this emergent Schrödinger equation, we can deduce a series of quantum behaviours. It's important to remark that the space-time sensible mass m_{ST} in the Schrödinger equation of free particles coincide with the inertial mass m of free particles.Since we only discuss non-relativistic quantum mechanics in the followings, we don't need to distinguish m_{ST} from m any more. From $|\Psi|^2 = e^{2R}$ and $\nabla R = \frac{1}{2\Re}\vec{u}$, we have

$$\vec{u} = \Re\frac{\nabla|\Psi|^2}{|\Psi|^2} \tag{III.27}$$

which leads to Born rule $\rho = |\Psi|^2$. $\rho(x,t)$ is the probabilistic density of particles in coordinate x at time t. The Born rule is a law of quantum mechanics which gives the probability that a measurement on a quantum system will yield a given result, which became a fundamental ingredient of Copenhagen interpretation. In this paper, we are attempt to suggest an interpretation of Born rule according to the StIP, which can provide a materialistic point of view for wave function. Emergent from random impacts of space-time, it's absolutely necessary that wave function is complex. If wave function were a real sine or cosine function[13], according to $\rho = |\Psi|^2$, the probabilistic density of a free particle with definite momentum would oscillate periodically which violates the isotropy of physical space.

B. Physical meanings of potential functions R and I

Substituting $\Psi = e^{R+iI}$ into $\frac{\partial\Psi}{\partial t} = i\Re\nabla^2\Psi$, we equalise the real and imaginary part separately as

$$\partial_t R = -\Re(2\nabla R\cdot\nabla I + \nabla^2 I) \tag{III.28}$$

$$\partial_t I = \Re[(\nabla R)^2 - (\nabla I)^2 + \nabla^2 R] \tag{III.29}$$

Combining with previous result $\rho = |\Psi|^2 = e^{2R}$, we have

$$\partial_t \rho = 2\rho \partial_t R \tag{III.30}$$

$$\nabla \rho = 2\rho \nabla R \tag{III.31}$$

The differential equation of potential R can be turned into

$$\partial_t \rho = -2\Re \nabla \cdot (\rho \nabla I) \tag{III.32}$$

With $\nabla I = \frac{1}{2\Re}\vec{v}$, the differential equation of potential R is equivalent to the equation of continuity

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \tag{III.33}$$

Noticing that the classical momentum of particle is $m\vec{v} = \hbar \nabla I$, we find that the differential equation of potential I goes to

$$\partial_t(\hbar I) + \frac{(\nabla(\hbar I))^2}{2m} - \hbar\Re[(\nabla R)^2 + \nabla^2 R] = 0 \tag{III.34}$$

Comparing with the Hamilton-Jacobi equation from classical mechanics [14, 15] as

$$\partial_t S + \frac{(\nabla S)^2}{2m} + V(x) = 0 \tag{III.35}$$

which is particularly useful in identifying conserved quantities for mechanical systems. There are two crucial remarks: Firstly, potential function I is proportional to the Hamilton-Jacobi function S as $S = \hbar I$. Secondly, for a free particle, the inference of space-time can be summed up to the space-time potential

$$V_{ST} = -\hbar\Re[(\nabla R)^2 + \nabla^2 R] \tag{III.36}$$

where the space-time potential V_{ST} will play the same role of potential V in the Hamilton-Jacobi equation. The space-time potential V_{ST} vanishes in the classical limit $\hbar = 0$, which is equivalent to $V = 0$ for free particles in classical mechanics. The quantum effect, which corresponding to nonzero $\hbar$, now is the nature result of the existence of the space-time potential V_{ST}, induced by space-time interaction. In principal, the moving of free particle can be described precisely by the space-time potential V_{ST} as

$$m\frac{d^2\vec{x}}{dt^2} = -\nabla V_{ST} = \hbar\Re\nabla[(\nabla R)^2 + \nabla^2 R] \tag{III.37}$$

C. Space-time random motion of charged particles in electromagnetic field

According to the StIP, electromagnetic field only serves as an external potential, which itself is not affected by random impacts of space-time. In a electromagnetic field $(\vec{E}, \vec{B})$, the charged particle will experience a Lorentz force $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$. Therefore, the average acceleration of charged particles will be

$$\vec{a} = e(\vec{E} + \vec{v} \times \vec{B})/m \tag{III.38}$$

where m is the inertial mass of charged particle and e is the charge. Based on the space-time principle, we are able to derive the equation of motion of charged particle in electromagnetic field, which is finally shown to be Schrödinger equation in electromegnatic field, which is

$$i\hbar\partial_t\Psi = \frac{1}{2m}(-i\hbar\nabla - \frac{e}{c}\vec{A})^2\Psi + e\phi\Psi \tag{III.39}$$

where the electromagnetic potential and the electromagnetic field are connected by

$$\vec{B} = \nabla \times \vec{A}, \vec{E} = -\partial_t\vec{A} - \nabla\phi. \tag{III.40}$$

We do not have average acceleration in the case absence of electromagnetic field, however, this is not the case when the particle have non-zero electric charge, moving in external electromagnetic field. Identifying the speed in the Lorentz force as the classical speed of random motion of particle in space-time, we have

$$\partial_t\vec{v} = e(\vec{E} + \vec{v} \times \vec{B})/m - (\vec{v} \cdot \nabla)\vec{v} + (\vec{u} \cdot \nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{III.41}$$

In the electromagnetic field, the equation of motion of charged particle becomes coupled non-linear partial differential equations as following

$$\frac{\partial\vec{u}}{\partial t} = -\Re\nabla(\nabla \cdot \vec{v}) - \nabla(\vec{u} \cdot \vec{v}) \tag{III.42}$$

$$\frac{\partial\vec{v}}{\partial t} = e(\vec{E} + \vec{v} \times \vec{B})/m - (\vec{v} \cdot \nabla)\vec{v} + (\vec{u} \cdot \nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{III.43}$$

In order to solve the coupled non-linear partial differential equations, we have to linearise it firstly. Let $\Psi = e^{R+iI}$ and notice that the canonical momentum of charged particle [16] is $\vec{p} = m\vec{v} + e\vec{A}/c$, we suppose

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{III.44}$$

$$\nabla I = \frac{1}{2\Re}(\vec{v} + \frac{e\vec{A}}{mc}) \tag{III.45}$$

In order to prove Eq.(III.39), we expand the first term of right side of Eq.(III.39) as

$$\frac{1}{2m}(-i\hbar\nabla - \frac{e}{c}\vec{A})^2\Psi = -\frac{\hbar^2\nabla^2}{2m}\Psi + \frac{e^2A^2}{2mc^2}\Psi + \frac{i\hbar e}{2mc}(\nabla\cdot\vec{A})\Psi + \frac{i\hbar e}{mc}\vec{A}\cdot(\nabla\Psi) \tag{III.46}$$

Substituting $\Psi = e^{R+iI}$, it leads to

$$-\frac{\hbar^2}{2m}[\nabla^2 R + i\nabla^2 I + (\nabla R + i\nabla I)^2]\Psi + \frac{e^2A^2}{2mc^2}\Psi + \frac{i\hbar e}{2mc}(\nabla\cdot\vec{A})\Psi + \frac{i\hbar e}{mc}(\vec{A}\cdot(\nabla R + i\nabla I))\Psi \tag{III.47}$$

With vector formulas

$$\nabla(\vec{A}\cdot\vec{B}) = \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A}) + (\vec{A}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{A} \tag{III.48}$$

$$\nabla(\nabla\cdot\vec{A}) = \nabla\times(\nabla\times\vec{A}) + \nabla^2\vec{A} \tag{III.49}$$

and Eq.(III.44), we will obtain

$$\nabla\times\vec{u} = 0 \tag{III.50}$$

$$\nabla\times(\vec{v} + \frac{e\vec{A}}{mc}) = 0 \tag{III.51}$$

Straightforwardly, we have

$$i\hbar(\partial_t R + i\partial_t I) = -\frac{\hbar^2}{2m}[\nabla^2 R + i\nabla^2 I + (\nabla R + i\nabla I)^2] + \frac{e^2A^2}{2mc^2} + \frac{i\hbar e}{2mc}(\nabla\cdot\vec{A}) + \frac{i\hbar e}{mc}(\vec{A}\cdot(\nabla R + i\nabla I)) + e\phi \tag{III.52}$$

Now, let's prove that the real and imaginary parts are separately equaled as

$$\partial_t I = \frac{\hbar}{2m}(\nabla^2 R + (\nabla R)^2 - (\nabla I)^2) - \frac{e^2\vec{A}^2}{2mc^2} + \frac{e}{mc}(\vec{A}\cdot(\nabla I)) - \frac{e\phi}{\hbar} \tag{III.53}$$

$$\partial_t R = -\frac{\hbar}{2m}(\nabla^2 I + 2(\nabla R)\cdot(\nabla I)) + \frac{e}{2mc}(\nabla\cdot\vec{A}) + \frac{e}{mc}\vec{A}\cdot(\nabla R) \tag{III.54}$$

Taking the gradient from both sides and the definitions $\vec{B} = \nabla \times \vec{A}, \vec{E} = -\partial_t \vec{A} - \nabla \phi$, we have reproduced the Eq.(III.42). Therefore, we have proved that both sides of Eq.(III.42) are at most different from a zero gradient function. It's important to notice that the choices of electromagnetic potentials are not completely determined. It allows a gauge transformation [17]

$$\vec{A}' = \vec{A} + \nabla \Lambda \tag{III.55}$$

$$\phi' = \phi - \partial_t \Lambda \tag{III.56}$$

For any function $\Lambda(\vec{x}, t)$, the electromagnetic field is invariant. Therefore, the corresponding wave function cannot change essentially, at most changing a phase factor. Given $\psi' = \psi e^{\frac{ie\Lambda}{\hbar c}}$, Schrödinger equation of charged particle in electromagnetic field is invariant, i.e., $U(1)$ gauge symmetry. By choosing the function $\Lambda(\vec{x}, t)$ properly, we are able to eliminate the redundant zero gradient function. So we have proved Eq.(III.39) at the end.

D. Stationary Schrödinger equation from StIP

Let's take a hydrogen atom as an example. Given $\vec{A} = 0$ and $\phi = -\frac{e}{4\pi\epsilon_0 r}$ for a hydrogen atom, the stationary solution of Eq.(III.39) is

$$E\Psi = \frac{1}{2m}(-i\hbar\nabla)^2\Psi - \frac{e^2}{4\pi\epsilon_0 r}\Psi \tag{III.57}$$

of which ground state wave function is $\Psi(r, \theta, \varphi) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$ and $a = 5 \times 10^{-11} m$ is the Bohr radius. Corresponding to the classical speed from Eq.(III.24), it is easy to show that the classical speed of particles must be zero in stationary states. Within the framework of StIP, we should interpret the stationary states from quantum mechanics as a space-time random motion with zero classical speed. Once we have all the stationary states, we will get the general solution by linear superposition. Therefore, we are going to derive stationary Schrödinger equation from classical speed $\vec{v} = 0$, which can provide a clear physical picture of StIP. Moreover, when $|\vec{v}|$ is large and close to speed of light c, the generalisation of this framework is clear and will be explained in a further project.

The trajectory of random motion of particle can be understood as the superposition of classical path and fluctuated path. During time interval $\triangle t$, there are two contributions to the trajectory as

$$\delta \vec{x} = \vec{u}(\vec{x}, t)\triangle t + \triangle \vec{x} \tag{III.58}$$

of which distribution satisfies $\varphi(\triangle\vec{x}) = \varphi(-\triangle\vec{x})$ and $\int\varphi(\triangle\vec{x})d(\triangle\vec{x}) = 1$. The space-time coefficient reads

$$\Re = \frac{1}{2\triangle t}\int(\triangle\vec{x})^2\varphi(\triangle\vec{x})d(\triangle\vec{x}) \tag{III.59}$$

The probabilistic density $\rho(x,t)$ evolves as

$$\rho(\vec{x}, t+\triangle t) = \int\rho(x-\delta\vec{x}, t)\varphi(\triangle\vec{x}))d(\triangle\vec{x}) \tag{III.60}$$

Expanding Taylor series of both sides, we have

$$\partial_t\rho = -\nabla\cdot(\rho\vec{u}) + \Re\nabla^2\rho \tag{III.61}$$

which is consistent with Fokker-Planck equation. In any external potential $V(\vec{x})$, there are two contributions to the changing of average speed. One is from random impacts of space-time, another one is from acceleration provided by external potential. Therefore, the average speed will evolve during time interval $\triangle t$ as

$$\vec{u}(\vec{x}, t+\triangle t) = \frac{\int(\vec{u}(\vec{x}-\delta\vec{x}, t) - \triangle t\nabla V(\vec{x}-\delta\vec{x})/m)\rho(x-\delta\vec{x}, t)\varphi(\triangle\vec{x}))d(\triangle\vec{x})}{\int\rho(x-\delta\vec{x}, t)\varphi(\triangle\vec{x}))d(\triangle\vec{x})} \tag{III.62}$$

the denominator of eq. $III.62$ is the normalisation factor of the probability distribution. Expanding Taylor series of both sides, we obtain

$$\frac{d\vec{u}}{dt} = -\nabla V + \Re m(\frac{\nabla^2(\rho\vec{u})}{\rho} - \vec{u}\frac{\nabla^2\rho}{\rho}) \tag{III.63}$$

With the condition of stationary state $\partial_t\rho = 0$, it goes to

$$\vec{u} = \Re\frac{\nabla\rho}{\rho} \tag{III.64}$$

$$\partial_t\vec{u} = 0 \tag{III.65}$$

It's important to notice that

$$\frac{d\vec{u}}{dt} = \partial_t\vec{u} + (\vec{u}\cdot\nabla)\vec{u} \tag{III.66}$$

Therefore, given the condition of stationary state, we are able to get

$$-2m\Re^2\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} + V(x) = Const. \tag{III.67}$$

We can prove this constant is exactly the average energy of particle

$$E = \int\rho(\frac{1}{2}mu^2 + V)d^3x \tag{III.68}$$

Now, we have derived

$$-2m\Re^2\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} + V(x) = E \tag{III.69}$$

$$\psi = \sqrt{\rho}e^{-iEt/\hbar} \tag{III.70}$$

Let $\Re = \frac{\hbar}{2m}$ once again, we arrive at the stationary Schrödinger equation

$$-\frac{\hbar^2\nabla^2}{2m}\psi + V\psi = i\hbar\partial_t\psi \tag{III.71}$$

IV. MINIMAL MASS FROM STIP

A. Existence of minimal mass

According to analysis in section 2, we know each collision of QST and the particle will change the action of the particle exactly by one $\hbar$. Let us investigate the motion of the particle in the rest frame before collision. If there are no lower bound of a mass of particle, then in the period of a collision Δt, the particle accquired an energy $\Delta E = \frac{\hbar}{\Delta t}$. It will only can be transformed as the kinetic energy, hence we have:

$$\frac{\hbar}{\Delta t} = \frac{1}{2}mv^2,$$

taking m goes to zero will end up with a infinity velocity. This obviously spoils the vacuum speed of light principle. By introducing the Lorentz transformation as in special relativity, the motion of the particle is controlled as

$$\left(\frac{\hbar}{\Delta t} + m_0c^2\right)^2 = m_0^2c^4 + \left(\frac{m_0}{\gamma(v)}\right)^2 v^2c^2 \tag{IV.1}$$

$$\gamma(v) = \sqrt{1 - v^2/c^2}.$$

When the velocity close to c, the vacuum light speed, the equation $(IV.1)$ never can be satisfied. This means relativity can not cure the issue.

The way out, is to introduce the space-time sensible minimal mass $\bar{m}_{ST}$. If and only if the mass $m > \bar{m}_{ST}$, the object under space-time collision can be seen as a particle in common sense, otherwise the collision will never happen and the object is "invisible" by QSTs.

B. Evaluation of the minimal mass

1. First estimation

If we have a mass $m < \bar{m}_{ST}$ particle in space-time, by definition it will not be detected by the QST all the way, all the time. Hence it will go along a straight line, with a constant speed. According to the StIP, each collision results an changing of action of the particle an $\hbar$. Let's assume the collision time is δt and during it the particle experiences a displacement δx, a changing of energy δE, and a changing of momentum δP, so we obtain:

$$\hbar = \delta E \delta t = \delta P \delta x. \tag{IV.2}$$

There is cosmological microwave backgroud(CMB) radiation in the universe everywhere, which makes our empty space-time have temperature 2.73K. Let's assume the energy density in empty space-time is ρ. To result a random motion without friction for a particle under StIP, the space-time sensible mass has the lower bound $\bar{m}_{ST}$, which satisfies:

$$\bar{m}_{ST} c^2 = \rho \frac{4\pi(\delta x)^3}{3}. \tag{IV.3}$$

Combining eq.$(IV.2)$ and eq.$(IV.3)$, we can obtain:

$$\bar{m}_{ST} = \left(\frac{4\pi\rho\hbar^3}{3c^5} \right)^{1/4}. \tag{IV.4}$$

Nowadays the experiment value of the energy density of CMB is $\rho = 4.005 \times 10^{-14} J/m^3$, substitute into $(IV.4)$, we obtain $\bar{m}_{ST} \simeq 2.117 \times 10^{-39} kg$.

2. Second estimation

The space-time sensible minmial mass, can have an intuitive understanding, that a rest particle with mass $\bar{m}_{ST}$ will have its De Broglie frequency $\bar{\omega}$ and satisfy:

$$\bar{m}_{ST} c^2 = \hbar\bar{\omega}. \tag{IV.5}$$

When the frequency $\bar{\omega}$ of the particle is smaller than the characteristic frequency of CMB, its signal will be erased by the noise of the CMB. The characteristic frequency of CMB ω is

$$kT = \hbar\omega \tag{IV.6}$$

from which we can estimate the minimal mass that can be detected by QSTs will be

$$\bar{m}_{ST} = \frac{kT}{c^2} \simeq 4 \times 10^{-40} kg. \tag{IV.7}$$

The conclusion now we have for the moment is: if the mass of a particle $m < \bar{m}_{ST}$, there will be no interaction between QSTs and the particle, so no randam motion for the particle will be emergent, the particle will move in speed c along a straight line.

V. FROM STIP TO UNCERTAINTY PRINCIPLE

We believed that the root of Heisenberg's uncertainty principle is the random impacts of space-time, which origins from that $d\vec{x}/dt$ is not well defined according to the StIP. One has to notice immediately that the momentum of particle can not be defined as $\vec{p} = md\vec{x}/dt$. It's reasonable to define momentum as

$$\vec{p} = mD\vec{x} = m\vec{v} + m\vec{u} \tag{V.1}$$

where

$$\vec{u} = \Re \frac{\nabla \rho}{\rho} \tag{V.2}$$

For any random variable O, its average value is $< O >= \int O\rho(x)dx$. Multiplying $m\rho$ into both sides and integrating x, we can obtain the covariance between x and u_x as

$$\sigma(x, u_x) =< (x- < x >)(u_x- < u_x >) >= -\Re \tag{V.3}$$

where covariance is a measure of how much two random variables change together. The sign of the covariance therefore shows the tendency in the linear relationship between the variables. For any two random variables A and B, according to Cauchy-Schwarz inequality $|\sigma(A, B)| \leq \sigma(A)\sigma(B)$, we have

$$\sigma(x)\sigma(u_x) \geq \Re = \hbar/2m \tag{V.4}$$

where the corresponding standard errors are

$$\sigma(x) = \sqrt{< x^2 > - < x >^2} \tag{V.5}$$

$$\sigma(u_x) = \sqrt{< u_x^2 > - < u_x >^2} \tag{V.6}$$

We have proved the uncertainty relations between coordinate and fluctuated speed of particles in random motion. Furthermore, the fluctuations of momentum come from

$$\sigma^2(p) = m^2(\sigma^2(v) + \sigma^2(u)) \tag{V.7}$$

Substituting $\sigma(p) \geq m\sigma(u)$, we arrive

$$\sigma(x)\sigma(p_x) \geq \hbar/2 \tag{V.8}$$

From this proof, we can interpret Heisenberg's uncertainty principle as the uncertainty relations between coordinate and fluctuated speed of particles in random motion. Historically, in 1927, Kennard had given the first mathematical proof of uncertainty principle [18]. In the following year, Weyl proved uncertainty principle independently [19]. We should remark that the uncertainty principle has been confused with a somewhat similar effect in physics, called the observer effect, which notes that measurements of certain systems cannot be made without affecting the systems. From recent experiments in 2012 [20, 21], the original interpretation of Heisenberg such an observer effect at the quantum level as a physical "explanation" of quantum uncertainty is not satisfied. It has been since become clear, however, that the uncertainty principle must be inherent in the properties of all particles in space-time. In this paper, we show that the uncertainty principle is deeply rooted from random impacts of space-time. Within the framework of non-relativistic quantum mechanics, the mass of particle is the only intrinsic property that is sensible in space-time.

VI. FROM STIP TO PATH INTEGRAL

Historically, the basic idea of the path integral formulation can be traced back to Norbert Wiener, who introduced the Wiener integral for solving problems in stochastic process [3]. This idea was extended to the use of the Lagrangian in quantum mechanics by P. A. M. Dirac in his 1933 paper [22].The complete method was developed in 1948 by Richard Feynman [23]. The path integral formulation of quantum mechanics is a description of quantum theory which generalises the action principle of classical mechanics. It replaces the classical notion of a single, unique trajectory for a system with a sum, or functional integral, over an infinity of possible trajectories to compute a quantum amplitude. Although we only investigated non-relativistic quantum mechanics in this paper, it is worthy to remark that the path integral formulation was very important for the development of quantum field theory[24]. The advantages of the path integral formulation mostly come from putting space and time on the equal footing, which is convenient to generalise in the relativistic theory. However, the regulator of path integral have caused infamous troubles for the divergence in quantum field theory, which leads to the procedure of renormalization. Within the framework of StIP, all the properties of random motion particle are finite so that we are able to construct a theory without divergence from beginning. In other words, all the quantum behaviours of particles are emergent from the statistical description of stochastic process.

A. Path integral of free particle and space-time interaction coefficient

There are two kinetic variables with random motion particle in space-time, which are classical speed $\vec{v}$ and fluctuated speed $\vec{u}$. The corresponding kinetic equations are

$$\frac{\partial \vec{u}}{\partial t} = -\Re\nabla(\nabla \cdot \vec{v}) - \nabla(\vec{u} \cdot \vec{v}) \tag{VI.1}$$

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v} \cdot \nabla)\vec{v} + (\vec{u} \cdot \nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{VI.2}$$

Setting $\Psi = e^{R+iI}$, we are able to linearise as

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{VI.3}$$

$$\nabla I = \frac{1}{2\Re}\vec{v} \tag{VI.4}$$

which leads to

$$\frac{\partial \Psi}{\partial t} = i\Re\nabla^2\Psi \tag{VI.5}$$

During an infinite small time interval ϵ, the solution can be written in terms of integrals as

$$\Psi(x, t + \epsilon) = \int G(x, y, \epsilon)\Psi(y, t)dy \tag{VI.6}$$

which represents the superposition of all the possible paths from y to x. The critical observation of Feynman is the weight factor $G(x, y, \epsilon)$ will be proportional to $e^{iS(x,y,\epsilon)/\hbar}$, where $S(x, y, \epsilon)$ is the classical action of particle as

$$S(x, y, \epsilon) = \int L(x, y, \epsilon)dt = \int (K - U)dt = (\bar{K} - \bar{U})\epsilon \tag{VI.7}$$

$\bar{K}$ and $\bar{U}$ are average kinetic energy and potential energy separately. In order to show the equivalence between path integral formulation and the space-time interacting picture, we should derive our basic kinetic equations from the postulation of path integral $G(x,y,\epsilon) = Ae^{iS(x,y,\epsilon)/\hbar}$. For a free particle in space-time, one has $\bar{U}=0$,$\bar{L}=\frac{m}{2}(\frac{x-y}{\epsilon})^2$ and $S=\frac{m(x-y)^2}{2\epsilon}$, which leads to

$$\Psi(x,t+\epsilon) = A\int e^{\frac{im(x-y)^2}{2\hbar\epsilon}}\Psi(y,t)dy \tag{VI.8}$$

Setting $y-x=\xi$ and $\alpha=-\frac{im}{2\hbar\epsilon}$, it can be written in terms of

$$\begin{aligned}\Psi(x,t+\epsilon) &= A\int e^{-\alpha\xi^2}\Psi(x+\xi,t)d\xi \\ &= A\int e^{-\alpha\xi^2}(\Psi(x,t)+\xi\frac{\partial\Psi}{\partial x}+\frac{1}{2}\xi^2\frac{\partial^2\Psi}{\partial x^2}+\mathcal{O}(\xi^4))d\xi\end{aligned} \tag{VI.9}$$

With the properties of Gaussian integral

$$\int e^{-\alpha\xi^2}d\xi = \sqrt{\frac{\pi}{\alpha}} \tag{VI.10}$$

$$\int e^{-\alpha\xi^2}\xi d\xi = 0 \tag{VI.11}$$

$$\int e^{-\alpha\xi^2}\xi^2 d\xi = \frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}} \tag{VI.12}$$

we can obtain

$$\Psi(x,t+\epsilon) = A(\sqrt{\frac{\pi}{\alpha}}\Psi(x,t)+\frac{1}{4\alpha}\sqrt{\frac{\pi}{\alpha}}\frac{\partial^2\Psi}{\partial x^2}+\mathcal{O}(\alpha^{-\frac{5}{2}})) \tag{VI.13}$$

Setting $A=\sqrt{\frac{\alpha}{\pi}}$, we have

$$\Psi(x,t+\epsilon)-\Psi(x,t) = \epsilon\partial_t\Psi(x,t) = \frac{1}{4\alpha}\frac{\partial^2\Psi}{\partial x^2} \tag{VI.14}$$

From this integral, We observed that the most important contribution comes from $y-x=\xi\propto\sqrt{\epsilon}$, where the speed of particle is $\frac{y-x}{\epsilon}\propto\sqrt{\frac{\hbar}{m\epsilon}}$, we see here when $\epsilon\to 0$, the velocity divergent in order $\sqrt{1/\epsilon}$. The paths involved are, therefore continuous but possess no derivative, which are of a type familiar from study of stochastic process. With the isotropy of space, we have

$$\partial_t\Psi(\vec{x},t) = \frac{1}{4\alpha\epsilon}\nabla^2\Psi(\vec{x},t) \tag{VI.15}$$

Corresponding to the Eq. (VI.5), if one requires the equivalence between path integral formulation and StIP, there must be

$$i\Re = \frac{1}{4\alpha\epsilon} \tag{VI.16}$$

$$\Re = \frac{1}{4i\alpha\epsilon} = \frac{1}{4i(-\frac{im}{2\hbar\epsilon})\epsilon} = \frac{\hbar}{2m} \tag{VI.17}$$

Notice that $\Re$ is only an arbitrary parameter in the Eq.(III.21). The consistency between path integral and space-time interaction requires $\Re = \frac{\hbar}{2m}$. An arbitrary finite time interval Δt, can be cut into infinitely many pieces of infnitesimal time interval ϵ. And in each ϵ, the collisions leads to many different paths, one can pick one path and consectively another along the time direction, this will end up a path in Δt, sum over all possible paths in Δt gives an integration over path space, which is the celebrated historical summation or path integral. The method here can be straightforwardly generalised to the particle in the external potential as in following section.

B. Path integral of particle in an external potential and space-time interaction coefficient

In an external potential U, one has $\bar{U} = U(\frac{x+y}{2})$ and $\bar{L} = \frac{m}{2}(\frac{x-y}{\epsilon})^2$, which leads to the action

$$S = \frac{m(x-y)^2}{2\epsilon} - U(\frac{x+y}{2})\epsilon \tag{VI.18}$$

According to the path integral formulation, it must satisfy

$$\Psi(x, t+\epsilon) = A\int e^{\frac{im(x-y)^2}{2\hbar\epsilon} - \frac{iU(\frac{x+y}{2})\epsilon}{\hbar}}\Psi(y,t)dy = A\int e^{\frac{im(x-y)^2}{2\hbar\epsilon}}(1 - \frac{iU(\frac{x+y}{2})\epsilon}{\hbar})\Psi(y,t)dy \tag{VI.19}$$

To the lowest order of ϵ, it shows

$$U(\frac{x+y}{2})\epsilon = U(x + \frac{\xi}{2})\epsilon = U(x)\epsilon \tag{VI.20}$$

$$\Psi(x, t+\epsilon) = A\int e^{-\alpha\xi^2}(1 - \frac{iU(x)\epsilon}{\hbar})\Psi(x+\xi, t)d\xi \tag{VI.21}$$

From the properties of Gaussian integral in the previous section, we obtain

$$\Psi(x, t+\epsilon) = A(1 - \frac{iU(x)\epsilon}{\hbar})\sqrt{\frac{\pi}{\alpha}}\Psi(x,t) + A\frac{1}{4\alpha}\sqrt{\frac{\pi}{\alpha}}\frac{\partial^2\Psi}{\partial x^2} \tag{VI.22}$$

Setting $A = \sqrt{\frac{\alpha}{\pi}}$, we have

$$\partial_t\Psi(\vec{x}, t) = \frac{1}{4\alpha\epsilon}\nabla^2\Psi(\vec{x}, t) + \frac{1}{i\hbar}U\Psi(\vec{x}, t) \tag{VI.23}$$

To be consistent with the case of free particle, let's take $\Re = \frac{\hbar}{2m}$ which leads to

$$\partial_t\Psi(\vec{x}, t) = i\Re\nabla^2\Psi(\vec{x}, t) + \frac{1}{i\hbar}U\Psi(\vec{x}, t) \tag{VI.24}$$

Therefore we have derived the equation of motion from StIP.

VII. SUMMARIES AND CONCLUSIONS

The Copenhagen interpretation of quantum theory implies that we must renounce the possibility of describing an individual system in terms of an abstract concept of wave function. However, in this paper, we have proposed the StIP as an alternative interpretation which does not involve the abstract concept of wave function, but instead leads us to the materialistic point of view of particle and space-time. Within the broader framework of StIP, we are able to conceive of each individual system as being in a precisely definable state, whose changes with time are determined by definite laws. As long as the present form of Schrödinger equation is retained in the non-relativistic case, the physical results obtained with the StIP are precisely the same as those obtained with usual quantum mechanics. Theerfore, it is thus entirely possible that we can have a precise and objective description at the quantum level.

Within the framework of StIP, we have derived the space-time coefficient and propose the concept of space-time sensible mass in the non-relativistic case. More importantly, we have proved the existence of the minimum of space-time sensible mass, below which the particle cannot be sensible in space-time so that it will travel at the speed of light. When the inertial mass of particle is equal to the space-time sensible mass, we have proved that the equation of motion will be equivalent to Schrödinger equation. With the postulation of StIP, quantum behaviour will emerge from a statistical description of space-time random impacts on the experimental scale, including Schrödinger equation, Born rule, Heisenberg's uncertainty principle and Feynman's path integral formulation. It is shown that we can construct a physical picture distinct from Copenhagen interpretation, and reinvestigate the nature of space-time and reveal the origin of quantum behaviours from the realistic point of view.

I have a special thank to Dr. Xiaolu Yu and Dr. Jianfeng Wu for their insightful discussions and critical reading of the manuscript. I am also grateful to Dr. Peng Zhang, Dr. Mingwei Ma, Dr. Yuan Tian, Dr. Jinyan Liu, Yin Cui and Xiaohui Hu for helpful discussions.

---

[1] S. Weinberg, Lectures on Quantum Mechanics, Cambridge University, (2012)

[2] L. Erdos, Lecture Notes on Quantum Brownian Motion, arXiv: 1009.0843, (2010)

[3] N. Wiener, Differential space. J. Math and Phys. 58 , 131-174 (1923)

[4] J. Zinn-Justin, Quantum Field Theory and Critical Phenomena (4-th edition), Oxford University, (2002)

[5] S. Ross, A First Course in Probability (8th Edition), Pearson Prentice Hall, (2009)

[6] S. Weinberg, Cosmology, Oxford University, (2008)

[7] F. Reif, Fundamentals of Statistical and Thermal Physics, Waveland, (2009)

[8] S. Chandrashekhar, Rev. Mod. Phys., 15,1(1943).

[9] J. Doob, Stochastic processes. Wiley: New York, (1953).

[10] A. Einstein, Investigations on the theory of the brownian movement. Dover Edition(1956).

[11] G. E. Uhlenbeck and L. S. Ornstein, Rev. Mod. Phys., 36,823(1930)

[12] M. C. Wang and G. E Uhlenbeck, Rev. Mod. Phys., 17,323(1945).

[13] L. Kadanoff , Statistical Physics: statics, dynamics and renormalization. World Scientific Press, (2000).

[14] L. Landau and E. Lifshitz , Courses in theoretical physics, vol 1, Mechanics. Butterworth-Heinemann. (1976)

[15] H. Goldstein, Classical Mechanics (3rd Edition), Addison-Wesley, (2001)

[16] J. D. Jackson, Classical Electrodynamics (3rd Edition), Wiley, (1998)

[17] L. Landau and E. Lifshitz , Courses in theoretical physics, vol 3, Quantum Mechanics. Pergamon Press, (1977)

[18] E. H. Kennard, "Zur Quantenmechanik einfacher Bewegungstypen". Zeitschrift für Physik 44 (45): 326 (1927)

[19] H. Weyl, The Theory of Groups and Quantum Mechanics, Dover Edition, (1950)

[20] L. A. Rozema, A. Darabi, D. H. Mahler, A. Hayat, Y. Soudagar, and A. M. Steinberg, Phys. Rev. Lett. 109, 100404 (2012)

[21] L. A. Rozema, A. Darabi, D. H. Mahler, A. Hayat, Y. Soudagar, and A. M. Steinberg, Phys. Rev. Lett. 109, 189902 (2012)

[22] P.A.M. Dirac, Physikalische Zeitschrift der Sowjetunion, Band 3, Heft 1 , pp. 64-72Ň (1933)

[23] R. P. Feynman, Ph.D thesis. Princeton Press (1942).

[24] S, Weinberg, The Quantum Theory of Fields, Volume 1: Foundations, Cambridge University, (2005)