A HEAT-POWER CYCLE FOR ELECTRICITY GENERATION FROM HOT WATER WITH NON-AZEOTROPIC MIXTURES

CHU-JUN GU¹ and LAN LIN²

¹Thermal Energy Research Institute, Tianjin University, Tianjin and ²Department of Power Engineering, Beijing Graduate School, North-China Institute of Electric Power, Qinghe, Beijing, China

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Abstract—A new heat-power cycle is introduced. This cycle is suitable for electricity generation using relatively low temperatures for hot water or other fluids. The cycle provides improved electricity generation over the Carnot and the Lorenz cycles. We suggest a practical plan for applying the new cycle to electricity generation at moderate temperatures.

INTRODUCTION

The binary fluid cycle system for electricity generation using relatively low-temperature heat is currently used widely. The traditional binary fluid cycle system is the Rankine cycle system with a single working fluid of low-boiling point. The Carnot cycle is an approximation of the Rankine cycle. Figure 1 shows the Carnot cycle a-b-c-d-a for hot-water power generation. In Fig. 1, hot-water releases heat with its temperature raised by the heat source at temperature T₁ to T₂. The cooling water absorbs heat while its temperature is changed from T₃ to T₄.

In a power-generation system, there is an optimal evaporation temperature for which the power output per unit mass of hot water is a maximum. With a minimum temperature difference ⁰Tₑ for heat exchange in the evaporator, we have

\[ Tₑ = Tₑ + ⁰Tₑ \]  

Thus, Tₑ is determined by Tₑ and

\[ Tₑ \sim (Tₑ, Tₑ)^{1/2} \]  

Generally, Tₑ is larger than Tₑ. Therefore, hot water with temperature Tₑ is not used to produce power but drained. If we consider only the limitation of the minimum temperature difference for heat exchange, the areas a-m-b-a and d-c-n-d in Fig. 1 are not used to produce power. Therefore, the efficiency of the Carnot cycle system is lower than that of the cycle which possesses the area n-a-m-c.

In 1984, a Lorenz power cycle with a working fluid consisting of a non-azeotropic mixture for power generation was suggested. Figure 2 shows the Lorenz cycle a-b-c-d-a. In the Lorenz cycle, the evaporation process a-b of a non-azeotropic mixture is not isothermal but rather has a variable temperature between Tₑ and Tₑ. Similarly, the condensation process has a variable temperature between Tₑ and Tₑ. For the Lorenz cycle, line a-b in Fig. 2 is parallel to line d-c.

Although the Lorenz cycle is similar to the Carnot cycle, it is not a perfect cycle. There is an optimal evaporation temperature

\[ Tₑ \approx -Tₑ + [2Tₑ(Tₑ + Tₑ)]^{1/2} \]  

Therefore, the hot water with temperature Tₑ is not used to produce power but drained; Tₑ is generally larger than Tₑ. Because of the claim that line a-b is parallel to line d-c in the Lorenz cycle, area h-g-h'-h(=limit of the area h-g-g'-h) in Fig. 2 is not used to produce power.

The following is an important question: can we find a new power cycle which possesses perfect performance for power generation for the given hot-water source temperature, the specified inlet temperature of cooling water and the given minimum temperature difference for heat exchange?
In 1986, Chu-Jun Gu suggested that a new heat-power cycle\(^2\) yields improved performance for power generation, as compared with the Carnot and Lorenz cycles under the same operating conditions. This cycle will now be described.

**A NEW HEAT-POWER CYCLE**

The new cycle is shown in Fig. 3. In the new cycle a–b–c–d–a, the line a–b represents the evaporation process with temperature increasing from \(T_a\) to \(T_b\); line c–d corresponds to condensation with the temperature decreasing from \(T_c\) to \(T_d\); line b–c represents isentropic expansion; and line d–a corresponds to isentropic compression.

Figure 4 shows a comparison of the Carnot and Lorenz cycles with the new cycle. For the given hot-water source temperature, the given inlet temperature for water condensation, and the given minimum temperature difference for heat exchange (which is determined by the optimal evaporation temperature), the work for the Carnot cycle is represented by area \(a'–b'–c'–d'–a'\) in Fig. 4, while the work for the Lorenz cycle is represented by the area \(a''–b''–c''–d''–a''\). The work for the new cycle is represented by the area a–b–c–d–a. It is apparent that the power output per unit mass of hot-water for the new cycle is much greater than for either the Carnot or the Lorenz cycle.
When the temperature differences for heat exchange are the same everywhere in the heat exchanger, this heat exchanger has been matched optimally and the resulting heat exchange is defined to be the optimal heat exchange. The irreversibly lost work is a minimum in the optimally matched heat exchanger. In the new cycle, both an optimally matched evaporator and condenser are used.

Figure 5 shows a new multistage cycle \( a_n-b_1-c_1-d_1-c_2-d_2-\ldots c_n-d_n-a_n \), where line \( a_n-b_1 \) represents an evaporation process with temperature changing from \( T_{an} \) to \( T_{b1} \), line \( b_1-c_1 \) represents an isentropic expansion process and line \( d_1-a_1 \) corresponds to an isentropic compression process. The condensation process includes multistage condensation processes \( c_1-d_1, c_2-d_2, \ldots, c_n-d_n \). The work of the new multistage cycle is represented by area \( a_n-b_1-c_1-d_1-a_n \), and this area is equal to the area \( a_n-b_1-c_1-d_1-a_n \), which is the area of one stage in the new cycle \( a_n-b_1-c_1-d_1-a_n \). It can be proved that

\[
W_n = \text{area } a_n-b_1-c_0-d_n-a_n - \text{area } c_1-c_0-d_1-c_1 - \text{area } c_2-d_1-d_2-c_2 - \ldots
- \text{area } c_n-d_{n-1} - d_n - c_n
= \text{area } a_n-b_1-c_0-d_n-a_n - \frac{1}{2} \text{area } a_0-c_0-d_n-a_0
- \text{area } a_n-b_1-c_1-d_n-a_n = W_i
= \frac{1}{2} (S_{b1} - S_{an}) (T_{an} - T_{dn} + T_{b1} - T_{c1}),
\]

(4)
where $W_n$ is the work for $n$ stages of the new cycle $a_n-b_1-c_1-d_1-c_2-d_2 \ldots c_n-d_n-a_n$, $W_1$ is the work for one stage of the new cycle $a_n-b_1-c_1-d_1-a_n$; $S_{b_1}, S_{a_n}$ are entropies at the points $b_1$ and $a_n$ in the cycle, respectively. $T_{an}, T_{dn}, T_{b_1}, T_{c_1}$ are the temperatures of the points $a_n, d_n, b_1,$ and $c_1$ in the cycle, respectively.

Therefore, $W_n$ is determined only by the characteristic points $b_1, c_1, d_n, a_n$ of the new cycle and has no relation to the number of stages used. This is an important character of the new cycle, which simplifies computations. For this reason, only one stage of the new cycle is analysed in this paper (cf. Figs. 3 and 4). The heat released per unit mass of hot-water is

$$Q_h = C_p A(T_f - T_o),$$

where $C_p$ is the (assumed constant) specific heat of water and $A$ is the unit-conversion coefficient. The flow rate of the working fluid in the binary fluid-cycle system is

$$G = \frac{Q_h}{(S_b - S_a)(T_a + T_b)}.$$  

The power output per unit mass of hot-water is

$$W_h = A C_p (T_f - T_c)(S_b - S_a)(T_a - T_c + T_b - T_o) / [(S_b - S_a)(T_a + T_b)]$$

$$= A C_p (T_f - T_c)(T_a - T_c + T_b - T_o) / (T_a + T_b),$$

where $\lambda_a = T_f / T_o$ represents the extent to which hot-water energy is utilized and is called the energy-utilization coefficient; $\lambda_b = T_f / T_o$ represents the temperature change of the working fluid during condensation and is called the temperature-variance coefficient for condensation; $\lambda_c = T_f / T_o$ represents the temperature change of the cooling water and is called the temperature coefficient of the cooling water.

The derivative of $W_h$ with respect to $\lambda_a$ is

$$\frac{\partial W_h}{\partial \lambda_a} = AC_p T_c (2[(\lambda_b - \lambda_a) - (\lambda_a - \lambda_c)] - (1 - \lambda_a^2) / (1 + \lambda_a^2).$$

If $T_a > T_c$, then $\lambda_a > \lambda_b > \lambda_c$, $\partial W_h / \partial \lambda_a < 0$, and the output of the new cycle increases as the temperature $T_a$ decreases. For $\partial W_h / \partial \lambda_a = 0$, $\lambda_a = \lambda_a^o = [2(\lambda_c + \lambda_b)]^{1/2} - 1$; and the corresponding temperature is

$$T_a^o = [2T_o(T_c + T_b)]^{1/2} - T_b;$$
when \( T_s < T_s^0 \),

\[
\frac{\partial W_g}{\partial \lambda_a} > 0. \tag{11}
\]

Equation (11) indicates that the power output of the new cycle decreases as \( T_s \) decreases for \( T_s < T_s^0 \). The power output \( W_g \) is maximized when \( T_s = T_s^0 \) and this maximum power output decrease as \( T_s^0 \) increases. Furthermore, \( T_s^0 \) increases as \( T_c \) increases and, therefore, \( T_c - T_d \) must be as small as possible for maximal power output. Figure 6 shows the temperature \( T_s^0 \) as a function of the temperature difference \( T_c - T_d \).

The first derivative of \( W_g \) with respect to \( \lambda_a \) is

\[
\frac{\partial W_g}{\partial \lambda_a} = AC_p T_0 (\lambda_a - 1)/(\lambda_a + 1) < 0. \tag{12}
\]

Equation (12) indicates that the power output always increases as \( T_c \) decreases. For an optimally-matching condenser, the very small temperature difference \( T_c - T_d \) will make the flow rate of cooling water so large that the required heat exchange in the condenser is difficult to obtain.

From Figs. 4 and 6, we can see that the temperature \( T_s \) is close to \( T_d \) when the condensation-temperature drop is sufficiently small, i.e., \( T_c - T_d < 10^\circ C \). Thus, the new cycle approximates the cycle \( a_0 - b - c_0 - a_0 \) in Fig. 4. Therefore, the power output of the new cycle is larger than that for all other possible cycles, for given temperatures of the hot-water source, and cooling water and a minimum temperature difference for the heat exchanger. This conclusion is also correct for power-generation systems using sensible heat sources of working fluids other than water.

**A METHOD FOR REALIZING THE NEW CYCLE**

Under the conditions of given pressure and mixture composition, non-azeotropic binary mixtures of miscible refrigerants condense and boil over a temperature range. Since the temperature variance between the boiling and dew points is not large, it is difficult to utilize the new cycle by using only a one-stage cycle. We now present a method which can be used to realize the new heat-power cycle by using a multistage cycle. The new multistage cycle is shown on the \( T-S \) diagram of Fig. 7. The equipment arrangement that may be used is shown in Fig. 8.
The method suggests that hot-water flows through several evaporators successively (the number of evaporators is equal to \( n \)) with the temperature varying from \( T_1 \) to \( T_n \). Non-azeotropic mixtures are turned into vapor in \( n \) sets of evaporators. The vapors of the mixtures, which enter \( n \) sets of turbines, ideally expand at constant entropy from \( b_1 \) to \( d_1 \), \( b_2 \) to \( d_2 \), \ldots, \( b_n \) to \( d_n \), respectively, to produce power, and the processes of condensation from \( d_1 \) to \( c_1 \), \( d_2 \) to \( c_2 \), \ldots, \( d_n \) to \( c_n \) occur when cooling waters enter \( n \) sets of condensers, respectively, to carry away the rejected heat. The condensates are returned to the entrances of \( n \) sets of evaporators by \( n \) sets of pumps, and the new multistage cycle repeats.

We have now concluded the thermodynamic calculation for the new multistage cycle with non-azeotropic binary mixtures R12/R142b. The evaporators and condensers have been matched optimally in the calculation.

The parameters chosen for the calculation are: (i) the temperature of hot-water source = \( T_{\text{hi}} = 90^\circ \text{C} \); (ii) the hot-water flow rate = \( G_\text{h} = 50T/\text{h} \); (iii) the temperature of the cooling water = \( T_{\text{hl}} = 20^\circ \text{C} \); (iv) the temperature difference for heat exchange in the evaporators = \( \Delta T_e = 10^\circ \text{C} \); (v) the temperature difference for heat exchange in the condensers = \( \Delta T_c = 10^\circ \text{C} \); (vi) the molar concentrations of mixtures are divided into four groups = \( X_{212} = 0.5, 0.6, 0.7, 0.8 \). The results for the new multistage cycle, the Lorenz cycle and the Carnot cycle are shown in Table 1. The number of stages for the new multistage cycle is equal to 12.
Table 1. Comparison of power output for the Carnot cycle, the Lorenz cycle and the new multistage cycle.

<table>
<thead>
<tr>
<th>Types of cycles</th>
<th>Working fluids</th>
<th>Power output per unit mass of hot water and increasing rate of power output</th>
<th>Units</th>
<th>Concentration $(X_{R12})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>Carnot cycle</td>
<td>R12</td>
<td>Power output per unit mass of hot water</td>
<td>kJ/T</td>
<td>7063.897</td>
</tr>
<tr>
<td>Carnot cycle</td>
<td>R142b</td>
<td>Power output per unit mass of hot water</td>
<td>kJ/T</td>
<td>7066.998</td>
</tr>
<tr>
<td>Lorenz/R142b</td>
<td></td>
<td>Increase of power output compared with the Carnot cycle R12</td>
<td>%</td>
<td>12.84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Increase of power output compared with the Carnot cycle R142b</td>
<td>%</td>
<td>12.78</td>
</tr>
<tr>
<td>The new cycle</td>
<td>R12/R142b</td>
<td>Power output per unit mass of hot water</td>
<td>kJ/T</td>
<td>14653.339</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Increase of power output compared with the Lorenz cycle</td>
<td>%</td>
<td>83.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Increase of power output compared with the Carnot cycle R12</td>
<td>%</td>
<td>107.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Increase of power output compared with the Carnot cycle R142b</td>
<td>%</td>
<td>107.09</td>
</tr>
</tbody>
</table>
CONCLUSION

In the binary fluid-cycle system for hot-water electricity generation, the power output per unit mass of hot-water obtained from the new cycle is greater than for the Carnot and Lorenz cycles.

For the given hot-water source temperature difference, given inlet temperature for condensation water, and given minimum temperature difference for the heat exchanger, the new heat-power cycle is the optimal, technically feasible heat-power cycle.

In the new heat-power cycle with non-azeotropic mixtures, the condensation temperature drop $T_c - T_d$ (cf. Fig. 4) should be as small as possible in order to produce the maximal power output. The new multistage cycle can be constructed with compound power generation system that is economic and technically feasible.

REFERENCES